

Exercise Sheet 6
CS 2210 Logic for Computer Scientists - Fall 2016
Solutions due: November 17, 2016 - 3:30 pm

Exercise 29 Identify all predicate symbols (with their arity) and all function symbols (with their arity) in all of the following formulas where s is a constant:

$$\begin{aligned} & \forall x(P(x) \wedge \neg \exists y(Q(g(x), s) \wedge \neg Q(y, f(y)))) \\ & \quad \forall x A(x, s) \vee \exists y(Q(y, x) \wedge \neg P(y, s)) \\ & \quad \exists y(\forall x R(x) \rightarrow \forall x(Q(y, p(x, z), s))) \\ & \quad \forall x \neg R(x) \vee \exists x Q(f(x), g(g(x))) \end{aligned}$$

Exercise 30 Give all terms in all of the formulas in Exercise 29.

Exercise 31 Give all subformulas of each of the formulas in Exercise 29.

Exercise 32 For each of the formulas in Exercise 29, determine which variables are bound and which are free. Based on your answer, also determine if the formula is closed or open.

Exercise 33 Give a structure for the formula

$$\exists x \forall y(Q(x, y) \rightarrow (\neg Q(y, x) \wedge R(f(y))))$$

Determine whether the structure you gave is a model of the formula.

Exercise 34 Give two structures for the following formula, one of which is a model for the formula, and the other is not a model for the formula.

$$\forall x(\text{Car}(x) \wedge \text{Human}(\text{driverOf}(x)) \rightarrow \neg \text{AutonomousCar}(x)) \wedge \exists y(\text{AutonomousCar}(y) \wedge \neg \text{Human}(\text{driverOf}(y)))$$

Exercise 35 Give two structures for the following formula, one of which is a model for the formula and the other is not a model for the formula.

$$\forall x \exists y(P(x) \wedge Q(x, y, m(x, y)))$$