

**Exercise Sheet 6**  
**CS 2210 Logic for Computer Scientists - Spring 2016**  
**Solutions due: April 12, 2016 - 9:30 am**

**Exercise 37** Identify all predicate symbols (with their arity) and all function symbols (with their arity) in all of the following formulas where  $s$  is a constant:

$$\begin{aligned} &\forall x(P(x) \rightarrow \exists y(Q(f(x), s) \vee \neg Q(y, y))) \\ &\quad \exists x A(x, s) \wedge \neg \exists y(Q(y, x) \vee \neg P(y, s)) \\ &\quad \forall y(\exists x R(x) \rightarrow \exists x(\neg Q(y, p(x, z), s))) \\ &\quad \exists x \neg R(x) \wedge \forall x Q(x, g(g(x))) \end{aligned}$$

**Exercise 38** Give all terms in all of the formulas in Exercise 37.

**Exercise 39** Give all subformulas of each of the formulas in Exercise 37.

**Exercise 40** For each of the formulas in Exercise 37, determine which variables are bound and which are free. Based on your answer, also determine if the formula is closed or open.

**Exercise 41** Give a structure for the formula

$$\forall x \forall y (Q(x, y) \rightarrow Q(y, x)).$$

Determine whether the structure you gave is a model of the formula.

**Exercise 42** Give two structures for the following formula, one of which is a model for the formula, and the other is not a model for the formula.

$$\forall x(\text{human}(x) \rightarrow \text{parentOf}(\text{fatherOf}(x), x)) \wedge \forall x \forall y(\text{orphan}(x) \wedge \text{parentOf}(y, x) \rightarrow \text{dead}(y))$$

**Exercise 43** Give two structures for the following formula, one of which is a model for the formula and the other is not a model for the formula.

$$\forall x \exists y (P(x) \wedge Q(s(x), y)).$$

**Exercise 44** Give two different models of the following formula where oh is a constant.

$$\forall x(\text{locatedIn}(\text{capitalOf}(x), x)) \wedge \text{locatedIn}(\text{capitalOf}(\text{oh}), \text{oh}))$$