Exercise Sheet 5
CS 2210 Logic for Computer Scientists - Spring 2016
Solutions due: March 15, 2016 - 9:30 am

Exercise 37 Show, using truth table, that $(A \land B) \land (A \rightarrow \neg B)$ is unsatisfiable.

Exercise 38 Is $((B \lor C) \lor A) \land (\neg A \land \neg B)$ satisfiable? If so, give one of its models.

Exercise 39 Transform $\neg((A \lor B) \land (C \lor D) \land (E \lor F))$ into CNF.

Exercise 40 Give a NNF, CNF, and DNF of the formula $\neg(I \lor \neg B) \lor \neg F$.

Exercise 41 Give a NNF, CNF, and DNF of the formula $\neg((A \land (B \land D)) \lor (A \lor F))$.

Exercise 42 (no hand-in) Show by structural induction: For any formula F (with all brackets written), we have $b(F) \leq c(F)$, where $b(F)$ is the number of all opening brackets in F, and $c(F)$ is the number of all connectives in F.

Exercise 43 (no hand-in) Show the following: For all formulas F_i $(i = 1, 2, 3)$, $F_1 \lor (F_2 \land F_3)$ and $(F_1 \lor E) \land (E \leftrightarrow (F_2 \land F_3))$ are equisatisfiable (E is a propositional variable not occurring in F_1, F_2, F_3).

Exercise 44 Give a complete tableau for the formula $(\neg p \land \neg q \land \neg r) \lor (p \land \neg q \land \neg r)$. Is the formula satisfiable or unsatisfiable?

Exercise 45 Determine if $((p \land q) \lor (p \land \neg q)) \land \neg(\neg r \land p)$ valid, satisfiable, or unsatisfiable **without** using a truth table.

Exercise 46 Modus Tollens hold if $(\neg Q \land (P \rightarrow Q)) \rightarrow \neg P)$ is a tautology. Show that this is indeed the case by using the tableaux algorithm.

Exercise 47 Show $\{A \rightarrow (B \rightarrow C)\} \models (A \rightarrow B) \rightarrow (A \rightarrow C)$ using the tableaux algorithm.

Exercise 48 Prove Theorem 2.6.8 part 2: "a formula F is a tautology if and only if there is a closed tableau for $\neg F$."
Hint: This needs less than two lines: try to reduce it to part 1 of the theorem.

Exercise 49 (no hand-in) For any formula F, let F' be the formula obtained from F by replacing all $\lor$ by $\land$, and by replacing all $\land$ by $\lor$. Furthermore, let $\overline{F}$ be obtained from F by replacing each occurrence of an atomic formula A in F by $\neg A$.
Example: For $F = (A \land B) \lor \neg C$, we have $F' = (A \lor B) \land \neg C$ and $\overline{F} = (\neg A \land \neg B) \lor \neg\neg C$; and $\overline{F}' = (\neg A \lor \neg B) \land \neg\neg C$.
Show by structural induction: $F \equiv \neg\overline{F}'$ for each formula F.