**Exercise Sheet 4**
**CS 2210 Logic for Computer Scientists - Spring 2017**
**Solutions due: April 14 2017 - 12:20 pm**

**Exercise 25** Identify all predicate symbols (with their arity) and all function symbols (with their arity) in all of the following formulas where $s$ is a constant:

$$\forall x A(x, s) \vee \exists y(Q(y, x) \wedge \neg P(y, s))$$
$$\forall x \neg R(x) \vee \exists x Q(f(x), g(g(x)))$$

**Exercise 26** Give all terms in all of the formulas in Exercise 25.

**Exercise 27** Give all subformulas of each of the formulas in Exercise 25.

**Exercise 28** For each of the formulas in Exercise 25, determine which variables are bound and which are free. Based on your answer, also determine if the formula is closed or open.

**Exercise 29** Give two structures for the following formula, one of which is a model for the formula, and the other is not a model for the formula.

$$\forall x\big(\mathrm{Car}(x) \wedge \mathrm{Human}(\mathrm{driverOf}(x)) \rightarrow \neg \mathrm{AutonomousCar}(x)\big) \wedge \exists y\big(\mathrm{AutonomousCar}(y) \wedge \neg \mathrm{Human}(\mathrm{driverOf}(y))\big)$$

**Exercise 30** Give two structures for the formula below so that one of the structure is a model of the formula and the other is not.

$$\forall x \forall y(Q(x, y) \rightarrow Q(y, x)).$$

**Exercise 31** What is $(\forall x(Q(x, y, z)[y/a])[x/b] \wedge \forall x(P(x, y)[y/x][x/a]))[z/x]$?

**Exercise 32** Show, using a tableau, that $\exists x(P(x) \wedge Q(x)) \models \exists x P(x) \wedge \exists y Q(y)$.

**Exercise 33** Show, using a tableau, that $Q(a) \wedge Q(b) \wedge \forall x(P(x) \wedge (Q(x) \rightarrow \neg P(x)))$ is unsatisfiable where $a, b$ are constants.

**Exercise 34** Show the following using a tableau:

$$\exists y(P(y) \wedge \neg S(y)) \wedge \forall y(\neg P(y) \vee Q(y)) \wedge \forall y(\neg Q(y) \vee (R(y) \wedge S(y))) \models \exists x(P(x) \wedge R(x))$$