

Exercise Sheet 4
CS 2210 Logic for Computer Scientists - Spring 2016
Solutions due: October 11 2016 - 3:30 pm

Exercise 16 Draw the following formulas as syntax trees and find all of their subformulas.

(a) $\neg(A \vee B) \rightarrow \neg C$

(b) $A \leftrightarrow \neg(B \wedge C)$

Exercise 17 If the truth assignment \mathcal{A} is given such that $\mathcal{A}(P) = 0$ and $\mathcal{A}(Q) = \mathcal{A}(R) = 1$, compute the truth value of the formula $\neg(P \vee \neg Q) \rightarrow \neg R$. That is, compute $\mathcal{A}(\neg(P \vee \neg Q) \rightarrow \neg R)$.

Exercise 18 Make the truth table for the formula from Exercise 17.

Exercise 19 Is the formula $(p \vee \neg(q \wedge r)) \wedge \neg p$ satisfiable? If so, give one of its models.

Exercise 20 Show that the following equivalences hold using any of the equivalence laws from the manuscript (Theorem 2.4.4).

(a) $\neg P \rightarrow \neg Q \equiv \neg(\neg P \wedge Q)$

(b) $\neg(P \wedge \neg Q) \rightarrow (R \rightarrow S) \equiv ((\neg P \vee Q) \wedge R) \rightarrow S$

Exercise 21 Determine if the following statements are true or false. Explain your answer.

(a) $A \vee B$ is a logical consequence of $A \wedge B$

(b) B is a logical consequence of $A \wedge \neg A$

[Hint: Recall Theorem 2.2.13. You can also use truth tables]

Exercise 22 Convert the following formulas into a negation normal form (NNF), a conjunctive normal form (CNF) and a disjunctive normal form (DNF).

(a) $(P \rightarrow (P \wedge \neg Q)) \rightarrow (P \vee \neg Q)$

(b) $(P \leftrightarrow \neg Q) \rightarrow \neg(P \vee \neg Q)$