

**Exercise Sheet 4**  
**CS 2210 Logic for Computer Scientists - Spring 2016**  
**Solutions due: February 23 2016 - 9:30 am**

**Exercise 27** Draw the following formulas as trees and find all of their subformulas.

- $((B \wedge F) \rightarrow \neg I)$
- $(\neg B \rightarrow \neg F)$

**Exercise 28** If the truth assignment  $\mathcal{A}$  is given such that  $\mathcal{A}(I) = 1$  and  $\mathcal{A}(B) = \mathcal{A}(F) = 0$ , compute the truth value of the formula  $\neg(I \vee \neg B) \vee \neg F$ . That is, compute  $\mathcal{A}(\neg(I \vee \neg B) \vee \neg F)$ .

**Exercise 29** Make the truth table for the formula from Exercise 28.

**Exercise 30** Give a model for  $\neg(p \wedge q) \vee \neg r$ .

**Exercise 31** Show the following.

1.  $A \wedge \neg A$  is unsatisfiable.
2.  $A \rightarrow \neg A$  is satisfiable.

**Exercise 32** Modus tollens is a logical inference in Aristotelian syllogistic logic and defined as follows:

If  $P$ , then  $Q$ .  
Not  $Q$ .  
Therefore, not  $P$ .

Express modus tollens in propositional logic.

**Exercise 33** Show, using truth tables, that modus tollens from Exercise 32 is valid/tautology.

**Exercise 34** For  $P$  the Datalog program given below where  $a$  and  $b$  are constants, determine  $v(P)$ .

$$\begin{aligned} & q(a) \\ & p(b) \\ & q(x) \rightarrow p(x) \\ & q(y) \wedge p(y) \rightarrow r(b) \end{aligned}$$

**Exercise 35** “What is the secret of your long life?” a centenarian was asked. “I strictly follow my diet: If I don't drink beer for dinner, then I always have fish. Any time I have both beer and fish for dinner, then I do without ice cream. If I have ice cream or don't have beer, then I never eat fish.”

Translate the “secrets” of the centenarian into propositional logic formulas, where  $B$  stands for *beer for dinner*,  $F$  for *fish for dinner* and  $I$  for *ice cream for dinner*.

**Exercise 36** Show that the centenarian in Exercise 35 always has beer for dinner.