Exercise Sheet 3
CS 2210 Logic for Computer Scientists - Spring 2017
Solutions due: March 13, 2017 - 12:20 pm

Exercise 11 For each of the following propositional logic formulas, list all of its subformulas.
(a) $\neg(A \vee B) \rightarrow \neg C$
(b) $A \leftrightarrow \neg(B \wedge C)$
(c) $\neg(P \vee \neg Q) \rightarrow \neg R$

Exercise 12 Let $\mathcal{A}$ be a truth assignment such that $\mathcal{A}(P) = 0$ and $\mathcal{A}(Q) = \mathcal{A}(R) = 1$. Compute $\mathcal{A}(\neg(P \vee \neg Q) \rightarrow \neg R)$, i.e., the truth value of $\neg(P \vee \neg Q) \rightarrow \neg R$.

Exercise 13 Using truth table, determine which of the following formulas is satisfiable. If so, give one of its models.
(a) $(p \vee \neg(q \wedge r)) \wedge \neg p$
(b) $(A \wedge B) \wedge (A \rightarrow \neg B)$
(c) $((B \vee C) \vee A) \wedge (\neg A \wedge \neg B)$

Exercise 14 Show that the folowing equivalences hold using any of the equivalence laws from the manuscript (Theorem 2.4.4).
(a) $\neg P \rightarrow \neg Q \equiv \neg(\neg P \wedge Q)$
(b) $\neg(\neg P \vee \neg Q) \rightarrow (\neg R \rightarrow S) \equiv (P \wedge \neg R) \rightarrow (Q \rightarrow S)$

Exercise 15 Determine if the following statements are true or false. Explain your answer.
(a) $A \wedge B$ is a logical consequence of $A \vee B$
(b) B is a logical consequence of $A \wedge \neg A$
(c) B is a logical consequence of $A \vee \neg A$

Exercise 16 Convert the following formulas into a negation normal form (NNF), a conjunctive normal form (CNF) and a disjuntive normal form (DNF).
(a) $(P \rightarrow (P \wedge \neg Q)) \rightarrow (P \vee \neg Q)$
(b) $(P \leftrightarrow \neg Q) \rightarrow \neg(P \vee \neg Q)$
(c) $\neg((A \wedge (B \wedge D)) \vee (A \vee F))$.

Exercise 17 Constructive dilemma is a rule of inference in propositional logic defined as follows:

If P, then Q.
If R, then S.
P or R.
Therefore, Q or S.

Express constructive dilemma in the form of logical entailment of a formula from a set of formulas, i.e., something like $\{F_1, \ldots, F_n\} \models G$.

Exercise 18 Show that constructive dilemma from Exercise 17 is a valid rule of inference using tableau algorithm.

Exercise 19 "What is the secret of your long life?" a centenarian was asked. "I strictly follow my diet: If I dont drink coffee for dinner, then I always have fish. Any time I have both coffee and fish for dinner, then I do without ice cream. If I have ice cream or dont have coffee, then I never eat fish."

Translate the "secrets" of the centenarian into propositional logic formulas, where C stands for *coffee for dinner*, F for *fish for dinner* and I for *ice cream for dinner*.

Exercise 20 Use the tableau algorithm to show that the centenarian in Exercise 19 always has coffee for dinner. [Hint: rephrase the question into a logical entailment problem].

Exercise 21 Determine if the formula $(\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$ satisfiable or unsatisfiable using tableau algorithm.

Exercise 22 Determine if $((p \wedge q) \vee (p \wedge \neg q)) \wedge \neg(\neg r \wedge p)$ valid, satisfiable, or unsatisfiable using tableau algorithm.

Exercise 23 Show $\{A \rightarrow (B \rightarrow C)\} \models (A \rightarrow B) \rightarrow (A \rightarrow C)$ using the tableaux algorithm.

Exercise 24 Use the tableaux algorithm to determine if $\{P \leftrightarrow (Q \rightarrow R))\} \models \neg(P \vee \neg(Q \vee R))$.