Exercise 11 A Datalog program P consists of the following rules where a, b, and c are constants.

$$r(b)$$
$$p(a, b)$$
$$q(b, c)$$
$$p(x, y) \rightarrow r(x)$$
$$q(x, y) \rightarrow r(y)$$
$$p(x, y) \wedge r(y) \wedge q(y, z) \rightarrow p(x, z)$$
$$r(x) \wedge r(y) \rightarrow q(y, x)$$

Give the Herbrand base B_P for P above. How many elements does I_P have? (Note that I_P is the set of all Herbrand interpretations of P).

Exercise 12 For the program P given in Exercise 11, compute the following:

(a) $T_P(\{r(a), r(b), p(a, b), p(b, a), q(a, b), q(b, c)\})$

(b) $T_P(B_P)$

Exercise 13 Determine if the following is true or false: for any Datalog program P, the Herbrand base B_P is always a pre-fixed point of T_P. Justify your answer.

Exercise 14 For the Datalog program P defined in Exercise 11,

(a) Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$.

(b) Determine whether $P \models_H q(c, c)$. Justify your answer.

Exercise 15 Let P be the Datalog program below where a, b are constants.

$$q(a)$$
$$p(b)$$
$$q(x) \rightarrow p(x)$$
$$q(y) \wedge p(y) \rightarrow r(b)$$

(a) Compute the two different Herbrand models of P. Justify your answer using the T_P operator.

(b) Determine whether $P \models_H r(a)$. Justify your answer.