Exercise 15 A Datalog program $P$ consists of the following rules where $a$ and $b$ are constants.

\[
\begin{align*}
p(a) \\
q(b) \\
q(x) \rightarrow q(x)
\end{align*}
\]

Give $B_P$ for $P$ above. How many elements does $I_P$ have? (Note that $I_P$ is the set of all Herbrand interpretations of $P$).

Exercise 16 A Datalog program $P$ consists of the following rules where $a, b, c$ are constants.

\[
\begin{align*}
p(a) \\
q(a, b) \\
q(b, c) \\
p(x) \rightarrow r(x) \\
r(x) \land q(x, y) \rightarrow r(y) \\
r(x) \land q(y, x) \rightarrow q(x, y)
\end{align*}
\]

Compute the following for the program $P$ above.

(a) $T_P(\{p(c), q(c, c)\})$

(b) $T_P(B_P)$

Exercise 17 With the program $P$ in Exercise 16, verify that $B_P$ is a pre-fixed point of $T_P$.

Exercise 18 Give three pre-fixed points and one fixed point of the $T_P$-operator for the Datalog program $P$ below where $a, b, c$ are constants.

\[
\begin{align*}
p(a, b) \\
q(c) \\
p(x, y) \rightarrow q(x)
\end{align*}
\]

Exercise 19 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program $P$ defined in Exercise 15.

Exercise 20 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program $P$ defined in Exercise 18.

Exercise 21 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program $P$ below where $a, b$ are constants.

\[
\begin{align*}
q(a) \\
p(b) \\
q(x) \rightarrow p(x) \\
q(y) \land p(y) \rightarrow r(b)
\end{align*}
\]

Note that in the fourth rule above, the head is ground (contains no variable).
Exercise 22 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program $P$ below where $a, b, c, d$ are constants.

\[
\begin{align*}
p(a, b) \\
p(b, c) \\
p(c, a) \\
p(d, d) \\
p(x, y) & \rightarrow q(x, y) \\
q(x, y) \land q(y, z) & \rightarrow q(x, z) \\
q(x, y) & \rightarrow r(x, y) \\
r(x, y) & \rightarrow r(y, x) \\
r(x, x) & \rightarrow t(x)
\end{align*}
\]

Exercise 23 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program $P$ below where $c, m, n$ are constants.

\[
\begin{align*}
\text{mOf}(x, y) & \rightarrow \text{pOf}(x, y) \\
\text{bOf}(x, y) \land \text{pOf}(y, z) & \rightarrow \text{uOf}(x, z) \\
\text{bOf}(c, m) \\
\text{mOf}(m, n)
\end{align*}
\]

Exercise 24 Show that the Datalog program from Example 1.1.1 (in the lecture manuscript) Herbrand-entails

\[
\text{grandmotherOf}(\text{ann}, \text{malia}).
\]

Exercise 25 (no hand-in – if coding helps you with the material) Write a computer program (you may choose your favorite language), which accepts as input graphs specified in the form of Example 1.1.5, and computes all $T_P \uparrow n$, where $P$ consists of all the non-fact Datalog rules from Example 1.1.5, plus the input graph encoded as facts.

Exercise 26 (no hand-in – give it a try if you’re math-minded) Given a Datalog program $P$, an interpretation $I \subseteq B_P$ is said to be supported if for every $A \in I$ there exists a rule $B_1 \land \cdots \land B_n \rightarrow A$ in ground($P$) with $\{B_1, \ldots, B_n\} \subseteq I$.

Show the following.

(a) An interpretation $I \in I_P$ is supported if and only if $I \subseteq T_P(I)$.

(b) The least Herbrand model of any program is supported.