Exercise Sheet 3
CS 2210 Logic for Computer Scientists - Spring 2016
Solutions due: February 9 2016 - 9:30 am

Exercise 15 A Datalog program P consists of the following rules where a and b are constants.

$$p(a)$$
$$q(b)$$
$$q(x) \to q(x)$$

Give B_P for P above. How many elements does I_P have? (Note that I_P is the set of all Herbrand interpretations of P).

Exercise 16 A Datalog program P consists of the following rules where a, b, c are constants.

$$p(a)$$
$$q(a, b)$$
$$q(b, c)$$
$$p(x) \to r(x)$$
$$r(x) \land q(x, y) \to r(y)$$
$$r(x) \land q(y, x) \to q(x, y)$$

Compute the following for the program P above.

(a) $T_P(\{p(c), q(c, c)\})$

(b) $T_P(B_P)$

Exercise 17 With the program P in Exercise 16, verify that B_P is a pre-fixed point of T_P.

Exercise 18 Give three pre-fixed points and one fixed point of the T_P-operator for the Datalog program P below where a, b, c are constants.

$$p(a, b)$$
$$q(c)$$
$$p(x, y) \to q(x)$$

Exercise 19 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program P defined in Exercise 15.

Exercise 20 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ forthe Datalog program P defined in Exercise 18

Exercise 21 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program P below where a, b are constants.

$$q(a)$$
$$p(b)$$
$$q(x) \to p(x)$$
$$q(y) \land p(y) \to r(b)$$

Note that in the fourth rule above, the head is ground (contains no variable).

Exercise 22 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program P below where a, b, c, d are constants.

$$\begin{aligned} &p(a,b) \\ &p(b,c) \\ &p(c,a) \\ &p(d,d) \\ p(x,y &\rightarrow q(x,y) \\ q(x,y) \wedge q(y,z) &\rightarrow q(x,z) \\ q(x,y) &\rightarrow r(x,y) \\ r(x,y) &\rightarrow r(y,x) \\ r(x,x) &\rightarrow t(x) \end{aligned}$$

Exercise 23 Compute $T_P \uparrow n$ for all $n \in \mathbb{N}$ and $T_P \uparrow \omega$ for the Datalog program P below where c, m, n are constants.

$$\begin{aligned} \mathrm{mOf}(x,y) &\rightarrow \mathrm{pOf}(x,y) \\ \mathrm{bOf}(x,y) \wedge \mathrm{pOf}(y,z) &\rightarrow \mathrm{uOf}(x,z) \\ &\mathrm{bOf}(c,m) \\ &\mathrm{mOf}(m,n) \end{aligned}$$

Exercise 24 Show that the Datalog program from Example 1.1.1 (in the lecture manuscript) Herbrand-entails

$$\mathrm{grandmotherOf}(\mathrm{ann}, \mathrm{malia}).$$

Exercise 25 (no hand-in – if coding helps you with the material) Write a computer program (you may choose your favorite language), which accepts as input graphs specified in the form of Example 1.1.5, and computes all $T_P \uparrow n$, where P consists of all the non-fact Datalog rules from Example 1.1.5, plus the input graph encoded as facts.

Exercise 26 (no hand-in – give it a try if you're math-minded) Given a Datalog program P, an interpretation $I \subseteq B_P$ is said to be *supported* if for every $A \in I$ there exists a rule $B_1 \wedge \cdots \wedge B_n \rightarrow A$ in ground(P) with $\{B_1, \ldots, B_n\} \subseteq I$.
Show the following.

(a) An interpretation $I \in I_P$ is supported if and only if $I \subseteq T_P(I)$.

(b) The least Herbrand model of any program is supported.