Exercise Sheet 2
CS 2210 Logic for Computer Scientists - Spring 2017
Solutions due: February 6, 2017 - 12:20 pm

Exercise 7 Evaluate the following substitutions. Which of them are ground?

(a) $(p(x,y,x) \wedge q(x,y,y) \wedge r(y,y) \rightarrow t(x))[x/a, y/b] = \ldots$

(b) $(q(a,x) \wedge p(x,y) \wedge q(y,a) \rightarrow r(y))[x/a][x/b] = \ldots$

(c) $(p(x,x) \wedge q(x,y) \rightarrow p(x,y))[y/b, x/b][y/c] = \ldots$

Exercise 8 Consider the following Datalog program P where a, b are constants.

$$q(a)$$
$$p(b)$$
$$p(x) \rightarrow q(x)$$
$$q(y) \wedge p(y) \rightarrow r(b)$$

(a) Give B_P (the Herbrand base of P, i.e., the set of all ground atoms that you could create from the predicates and constants in P). How many atoms are there in B_P?

(b) How many possible Herbrand interpretations of P are there in total?

(c) Give the grounding of the program P, i.e., ground(P).

(d) Give two distinct Herbrand models of the program P.

(e) Give two distinct Herbrand interpretations that are NOT a model of P. Justify why they are not a model of P.

(f) Compute the least Herbrand model of P. Show all the steps in your computation.

(g) Compute $T_P(\{q(a), p(a), p(b), q(b), r(a)\})$. Is $\{q(a), p(a), p(b), q(b), r(a)\}$ a model of P? Justify your answer using Theorem 1.3.11.

(h) Compute $T_P(\{q(a), p(b), q(b), r(b), r(a)\})$. Is $\{q(a), p(b), q(b), r(b), r(a)\}$ a model of P? Justify your answer using Theorem 1.3.11.

(i) Does $P \models_H q(b)$ hold? Justify your answer.

(j) Does $P \models_H r(a)$ hold? Justify your answer.

Exercise 9 Consider the following program P where a, b, c are constants.

$$p(a,b)$$
$$q(c)$$
$$p(x,y) \rightarrow q(y)$$

(a) Give B_P (the Herbrand base of P, i.e., the set of all ground atoms that you could create from the predicates and constants in P). How many atoms are there in B_P?

(b) Give the grounding of the program P, i.e., ground(P).

(c) How many possible Herbrand interpretations of P are there in total?

(d) Give two distinct Herbrand models of the program P.

(e) Give two distinct Herbrand interpretations that are NOT a model of the program P. Justify why they are not a model of P.

(f) Compute the least Herbrand model of P. Show all the steps in your computation.

(g) Compute $T_P(\{p(a,b), q(c), q(b), p(b,a)\})$. Is $\{p(a,b), q(c), q(b), p(b,a)\}$ a model of P? Justify your answer using Theorem 1.3.11.

(h) Compute $T_P(\{p(a,b), q(c), q(b), p(b,b)\})$. Is $\{p(a,b), q(c), q(b), p(b,b)\}$ a model of P? Justify your answer using Theorem 1.3.11.

(i) Does $P \models_H q(b)$ hold? Justify your answer.

(j) Does $P \models_H q(a)$ hold? Justify your answer.

Exercise 10 Consider the following program P where $0, 1, 2$ are constants.

$$
\begin{aligned}
s(0,1).\\
s(0,2).\\
s(x,y) &\rightarrow r(y,x)\\
r(x,y) &\rightarrow t(x)\\
t(x) \wedge t(y) &\rightarrow q(x,y)
\end{aligned}
$$

(a) Give B_P (the Herbrand base of P, i.e., the set of all ground atoms that you could create from the predicates and constants in P). How many atoms are there in B_P?

(b) Give the grounding of the program P, i.e., ground(P).

(c) How many possible Herbrand interpretations of P are there in total?

(d) Give two distinct Herbrand models of the program P.

(e) Give two distinct Herbrand interpretations that are NOT a model of the program P. Justify why they are not a model of P.

(f) Compute the least Herbrand model of P. Show all the steps in your computation.

(g) Compute $T_P(\{t(1)\})$. Is $\{t(1)\}$ a model of P? Justify your answer using Theorem 1.3.11.

(h) Compute $T_P(\{s(0,1), s(0,2), r(1,0), r(2,0), t(1), t(2), q(1,1), q(1,2), q(2,1), q(2,2), r(1,1)\})$. Is $\{s(0,1), s(0,2), r(1,0), r(2,0), t(1), t(2), q(1,1), q(1,2), q(2,1), q(2,2), r(1,1)\}$ a model of P? Justify your answer using Theorem 1.3.11.

(i) Does $P \models_H q(2,0)$ hold? Justify your answer.

(j) Does $P \models_H q(1,1)$ hold? Justify your answer.