Exercise 5 Let $L = (V, C, R)$ with $V = \{x, y\}$, $C = \{\text{barack, michelle, craig, malia}\}$ and $R = \{\text{motherOf}, \text{parentOf}, \text{grandmotherOf}\}$, all with arity 2.
Which of the Datalog facts (1) to (9) from Example 1.1.1 are atoms over $L$? Justify your answers.
(Note that the language $L$ is different from the language used in Example 1.1.1)

Exercise 6 Write a Datalog program which captures the following natural language sentences. Use the predicates: orphan (of arity 1), parentOf (of arity 2), dead (of arity 1), fatherOf (of arity 2), and the constants: harrypotter, and jamespotter.
(a) If somebody is an orphan, then all his parents are dead.
(b) Somebody’s father is also that person’s parent.
(c) Harry Potter is an orphan.
(d) James Potter is the father of Harry Potter.

Exercise 7 Evaluate the following substitutions. Which of them are ground?
(a) $(p(x, y, x) \land q(x, y, y) \land r(y, y) \rightarrow t(x))[x/a, y/b] = \ldots$
(b) $(q(a, x) \land p(x, y) \land q(y, a) \rightarrow r(y))[x/a][x/b] = \ldots$
(c) $(p(x, x) \land q(x, y) \rightarrow p(x, y))[y/b][y/c][x/b] = \ldots$

Exercise 8 Give the grounding of the Datalog program below where $a, b$ are constants.

\[
q(a) \\
p(b) \\
p(x) \rightarrow q(x) \\
q(y) \land p(y) \rightarrow r(b)
\]

Exercise 9 For the program in from Exercise 8,
(a) Give two distinct Herbrand models of the program.
(b) Give two distinct Herbrand interpretations that are NOT a model of the program.
Justify your answer.

Exercise 10 Give three distinct Herbrand models for the Datalog program $P$ consisting of the following rules where $a, b, c$ are constants.

\[
p(a, b) \\
q(c) \\
p(x, y) \rightarrow q(y)
\]