

**Exercise Sheet 2**  
**CS 2210 Logic for Computer Scientists - Spring 2016**  
**Solutions due: January 26, 2016 - 9:30 am**

**Exercise 6** Let  $L = (V, C, R)$  with  $V = \{w, y\}$ ,  $C = \{d, e\}$  and  $R = \{r, s\}$  where  $r$  has arity 1 and  $s$  has arity 2. Which of the following are atoms over  $L$ ? Which are ground atoms? Justify your answers.

- (a)  $d(w, w)$                       (b)  $r(d, e)$                       (c)  $s(w, w)$                       (d)  $r(y)$

**Exercise 7** Let  $L = (V, C, R)$  with  $V = \{x, y\}$ ,  $C = \{\text{barack, michelle, craig, malia}\}$  and  $R = \{\text{motherOf, parentOf, grandmotherOf}\}$ , all with arity 2.

Which of the Datalog facts (1) to (9) from Example 1.1.1 are atoms over  $L$ ? Justify your answers.

**Exercise 8** Write a Datalog program which captures the following natural language sentences.

- (a) If somebody is an orphan, then all his parents are dead.
- (b) Every orphan is a human being.
- (c) Somebody's father is also that person's parent.
- (d) Harry Potter is an orphan.
- (e) James Potter is the father of Harry Potter.

**Exercise 9** Give three distinct Herbrand interpretations for the following Datalog program, where  $a, b$  are constants.

$$\begin{aligned} & q(a) \\ & p(b) \\ & q(x) \rightarrow p(x) \\ & q(y) \wedge p(y) \rightarrow r(b) \end{aligned}$$

**Exercise 10** Evaluate the following.

- (a)  $(p(x, y, x) \wedge q(x, y, y) \wedge r(y, y) \rightarrow t(x))[x/a, y/b] = \dots$
- (b)  $(p(x) \wedge q(x) \rightarrow r(x))[x/c][x/d] = \dots$
- (c)  $(q(a, x) \wedge p(x, y) \wedge q(y, a) \rightarrow r(y))[x/a][x/b] = \dots$
- (d)  $(p(x, x) \wedge q(x, y) \rightarrow p(x, y))[y/b][y/c][x/b] = \dots$

**Exercise 11** Which of the substitutions in Exercise 10 are ground substitutions?

**Exercise 12** Give the grounding of the Datalog program from Exercise 9.

**Exercise 13** Give a Herbrand model for the Datalog program in Exercise 9.

**Exercise 14** Give three distinct Herbrand models for the Datalog program  $P$  consisting of the following rules.

$$\begin{aligned} & p(a, b) \\ & q(c) \\ & p(x, y) \rightarrow q(x) \end{aligned}$$