$\exists m \forall n\, P(n,m) \land \exists x \forall y\, \neg P(x,y)$

- $\exists m \forall n\, P(n,m)$
- $\exists x \forall y\, \neg P(x,y)$
- $\forall n\, P(n,a)$ m/a a is new
- $\forall y\, \neg P(b,y)$ x/b b is new

$P(b,a)$ n/b

$\neg P(b,a)$ y/a

n/c

$P(c,a)$

$\neg P(b,d)$

$\neg P(b) \land \neg Q(b)$ x/b

$\neg P(b)$

$\neg Q(b)$

because all branches are closed.

$\exists x \exists y (P(x) \lor Q(y)) \land \neg \exists x (P(x) \lor Q(x))$

is unsatisfiable.

$\exists x \exists y (P(x) \lor Q(y))$

$\models \exists x (P(x) \lor Q(x))$

Show: $\exists x \exists y (P(x) \lor Q(y)) \models \exists x (P(x) \lor Q(x))$

we show $\exists x \exists y (P(x) \lor Q(y)) \land \neg \exists x (P(x) \lor Q(x))$ is unsat.

$\exists x \exists y (P(x) \lor Q(y)) \land \neg \exists x (P(x) \lor Q(x))$

$\equiv \exists x \exists y (P(x) \lor Q(y)) \land \forall x \neg (P(x) \lor Q(x))$

$\equiv \exists x \exists y (P(x) \lor Q(y)) \land \forall x (\neg P(x) \land \neg Q(x))$

$\exists x \exists y (P(x) \lor Q(y)) \land \forall x (\neg P(x) \land \neg Q(x))$

$\exists x \exists y (P(x) \lor Q(y))$

$\forall x (\neg P(x) \land \neg Q(x))$

$\exists y (P(a) \lor Q(y))$ x/a a is new constant

$P(a) \lor Q(b)$ y/b b is new constant

$\neg P(a) \land \neg Q(a)$ x/a

$\neg P(a)$

$\neg Q(a)$

$P(a)$ | $Q(b)$